Non-unitarity effects in a realistic low-scale seesaw model

Michal Malinský,[*] Tommy Ohlsson,[†] and He Zhang[‡]

Department of Theoretical Physics, School of Engineering Sciences,
Royal Institute of Technology (KTH) – AlbaNova University Center,
Roslagstullsbacken 21, 106 91 Stockholm, Sweden

We analyze the structure of the non-unitary leptonic mixing matrix in the inverse seesaw model with heavy singlets accessible at the LHC. In this model, unlike in the usual TeV seesaw scenarios, the low-scale right-handed neutrinos do not suffer from naturalness issues. Underlying correlations among various parameters governing the non-unitarity effects are established, which leads to a considerable improvement of the generic non-unitarity bounds. In view of this, we study the discovery potential of the non-unitarity effects at future experiments, focusing on the sensitivity limits at a neutrino factory.

I. INTRODUCTION

One of the most intriguing open questions in particle physics nowadays is the origin of the unprecedentedly small neutrino masses and the peculiar leptonic mixing parameters observed in neutrino oscillation experiments. If neutrinos are Majorana particles, the seesaw approach [1, 2, 3, 4, 5, 6, 7] provides a natural link between the lepton sector observables and the dynamics underlying the breakdown of the lepton number. In the simplest schemes, the relevant scale typically falls into the 10^{11} GeV to 10^{14} GeV range, and thus seems never accessible to direct tests. However, there are options for a low-scale seesaw model, and in principle, the corresponding features of new physics can be probed in the forthcoming accelerator experiments such as those at the Large Hadron Collider (LHC).

The prospects of testing the origin of neutrino masses at colliders are determined by the scale of the underlying physics and one would appreciate it not to be far above the electroweak scale. If this is the case, a plethora of new effects such as the non-unitarity [8] of the Maki–Nakagava–Sakata (MNS) leptonic mixing matrix (imprinted into specific patterns of non-standard neutrino interactions, the enhancement of the lepton flavor violation (LFV) phenomena, etc.) can be within the reach of near future experimental facilities. This, in turn, provides a complementary strategy for unveiling the seesaw structure.

Unfortunately, within the popular type-I seesaw framework, this strategy is plagued by naturalness issues. Indeed, the requirement of reproducing the sub-eV neutrino masses tends to be incompatible with visible non-unitarity effects. The reason is the simplistic structure of the type-I seesaw formula, that in a convenient notation reads $m_\nu = FMF^T$. Here M is the right-handed (RH) neutrino mass scale and $F = M_D M^{-1}$ (with M_D denoting the Dirac neutrino mass matrix) corresponds to the structure governing the non-unitarity effects. One option of reconciling the TeV-scale M with the sub-eV light neutrino masses m_ν is to take F to be of the order of 10^{-5} which, however, leads neither to any appreciable non-unitarity effects nor to LHC signals. Alternatively, one can invoke a cancellation in the matrix structure of the seesaw formula [9, 10, 11, 12, 13], which is also not natural unless extra assumptions are made about the flavor structure of the model.

Concerning the other simple seesaw schemes, the situation in the type-III case is essentially identical to that in the type-I case, namely, there is no significant non-unitarity effect without fine-tuning the matrix structure [14, 15]. On the other hand, the type-II scheme with a light Higgs triplet offers distinctive features at the colliders as well as high-precision neutrino experiments and has been studied in great detail in e.g. Refs. [16, 17] and references therein. However, the leptonic mixing matrix is exactly unitary at the renormalizable level, since there is no RH sector the light states could admix with. Thus, none of the simple seesaw realizations of the Majorana neutrino masses provides a satisfactory framework accommodating both the collider phenomenology and the non-unitarity effects at an experimentally accessible level.

In this work, we therefore focus on the simplest inverse seesaw model [18], which shares all the virtues of the type-I seesaw scenario. In particular, it has the same predictive power concerning the non-unitarity as well as collider effects, yet providing a completely natural description of the sub-eV light neutrino masses. The key point is that in this framework the $B - L$ breaking mass insertion in the seesaw formula is decoupled from the RH neutrino mass scale, and thus, the light neutrino masses do not impose any stringent bounds on the size of the F parameters governing the interesting phenomenology. In this respect, the inverse seesaw scenario can be regarded as the simplest natural scheme accommodating RH neutrinos accessible to LHC whilst admitting complementary tests exploiting the would-be non-unitarity of the leptonic mixing matrix.

This work is organized as follows: In Sec. II, we comment in more detail on the structure of the inverse seesaw model and compare its parameter space to the conven-

---

[*]Electronic address: malinsky@kth.se
[†]Electronic address: tommy@theophys.kth.se
[‡]Electronic address: zhanghe@kth.se

tional type-I seesaw scheme. In Sec. III, we focus on the emergence and basic implications of the non-unitarity effects. The characteristic correlations between various non-unitarity effects in the context of some of the future experiments, in particular the neutrino factory and LHC direct searches, are studied in detail in Sec. IV.

II. THE INVERSE SEESAW MODEL

The inverse seesaw model [18] is an extension of the type-I seesaw scenario with three extra Standard Model (SM) gauge singlets S^α coupled to the RH neutrinos ν_R^α through the lepton number conserving couplings of the type $\overline{\nu_R^c} S$, while the traditional RH neutrino Majorana mass term is forbidden by extra symmetries. It is thus only through a dimensionful parameter μ in the self-coupling $\mu \overline{S} S^c$ the lepton number is broken and one can arrange μ to be arbitrarily small in a technically natural manner. The 9×9 mass matrix in the $\{\nu_L, \nu_R^c, S^c\}$ basis then reads

$$M_\nu = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & 0 & M_R \\ 0 & M_R^T & \mu \end{pmatrix} , \qquad (1)$$

where M_D and M_R are generic 3×3 complex matrices representing the Dirac mass terms in the ν_L-ν_R and ν_R-S sectors.[1] Without loss of generality, one can always choose a basis in which μ is real and diagonal: $\mu = \mathrm{diag}(\mu_1, \mu_2, \mu_3)$. The mass matrix M_ν can be diagonalized by means of a 9×9 unitary transformation

$$V^\dagger M_\nu V^* = \bar{M}_\nu = \mathrm{diag}(m_i, M_j^n, M_k^{\tilde{n}}) \qquad (2)$$

with $(i, j, k = 1, 2, 3)$, where m_i denote the masses of the left-handed neutrinos, while the RH neutrinos ν_R^α and the extra singlets S^α are almost maximally admixed into three pairs of heavy Majorana neutrinos $(n_j, \tilde{n}_j)$. Since n_j and $\tilde{n}_j$ have opposite CP parities and essentially identical masses M_j^n and $M_k^{\tilde{n}}$ (with a splitting of the order of μ), they can be regarded as components of three pseudo-Dirac neutrinos. Assuming $\mu \ll M_D < M_R$, the light neutrino Majorana mass term is given approximately by

$$m_\nu \simeq F \mu F^T , \qquad (3)$$

where, as in the type-I case, $F \equiv M_D (M_R^T)^{-1}$. Notice that the structure of the neutrino mass matrix is essentially identical to the type-I formula, i.e., it depends on two basic building blocks – the flavor structure of the $B - L$ breaking mass insertion (the RH neutrino Majorana mass matrix M in type-I and μ in the inverse seesaw setting) and the ratio F. Since in both cases, the

LFV, the non-unitarity effects as well as the LHC rates are driven only by F and the spectra of the heavy components involved in the charged currents, the relevant parameter spaces of these two scenarios are equivalent. From this perspective, the inverse seesaw enjoys a similar predictivity as the type-I case, but in a more realistic (yet experimentally interesting) regime.

III. THE NON-UNITARITY EFFECTS

The light neutrino mass matrix can be diagonalized by a unitary transformation U

$$U^\dagger m_\nu U^* = \bar{m}_\nu \qquad (4)$$

with $\bar{m}_\nu = \mathrm{diag}(m_1, m_2, m_3)$. In the standard (i.e., CKM-like) parametrization one has

$$U = P_\rho R_{23} P_\delta R_{13} P_\delta^{-1} R_{12} P_M , \qquad (5)$$

where R_{ij} correspond to the elementary rotations in the $ij = 23, 13$, and 12 planes (parametrized in what follows by three mixing angles $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$), $P_\delta = \mathrm{diag}(1, 1, e^{i\delta})$, and $P_M = \mathrm{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)$ contain the Dirac and Majorana CP phases, respectively. The $P_\rho = \mathrm{diag}(e^{i\rho_1}, e^{i\rho_2}, e^{i\rho_3})$ phases entering the charged currents are usually rotated away in the SM context (or in the regime the RH sector decouples) but must be kept in the current scenario. Even in the basis where the charged-lepton mass matrix is diagonal, U is only a part of the mixing matrix governing neutrino oscillations. Instead, one should look at the upper-left sub-block of the full 9×9 matrix V in Eq. (2)

$$V = \begin{pmatrix} V_{3\times3} & V_{3\times6} \\ V_{6\times3} & V_{6\times6} \end{pmatrix} . \qquad (6)$$

For M_R not far above the electroweak scale and a reasonably small μ, it is sufficient to consider the form of V at the leading order in F.[2] The full (non-unitary) MNS mixing matrix then reads [21, 22] (in the notation of Ref. [23]):

$$N \equiv V_{3\times3} \simeq \left(1 - \frac{1}{2} F F^\dagger\right) U , \qquad (7)$$

and the 3×6 block participating in the charged currents reads $K \equiv V_{3\times6} \simeq (0, F) V_{6\times6}$. These structures control all the observables of our further interest. The defining flavor eigenstates $\{\nu_L, \nu_R^c, S^c\}$ correspond to superpositions of the mass eigenstates $\{\hat{\nu}_L, n, \tilde{n}\}$, and the left-handed neutrinos entering the electroweak currents obey $\nu_L \simeq N\hat{\nu}_L + KP$, where $P = (n_1, .., n_3, \tilde{n}_1, .., \tilde{n}_3)$.

Using Eqs. (3) and (4), one can write $F = U\sqrt{\bar{m}_\nu}O\sqrt{\mu^{-1}}$ with O being a complex orthogonal matrix [24]. At the leading order in the non-unitarity of N, the entries of the unitary matrix U can be parametrized by the measured values of the leptonic mixing parameters, and thus, F depends only on m_1, O, and μ. As a simple example,[3] for $O = \mathbb{1}$, one obtains

$$\bar{m}_\nu = D\mu D^T \, , \tag{8}$$

where $D = \text{diag}(d_1, d_2, d_3)$ is a real and diagonal matrix acting as a *compensator* between the entries of μ and $\bar{m}_\nu$.

The charged current Lagrangian in the mass basis is

$$\mathcal{L}_{\text{CC}} = -\frac{g}{\sqrt{2}}\overline{\ell_{\text{L}}}\gamma^\mu \left(N\hat{\nu}_L + KP\right)W_\mu^- + \text{H.c.} \tag{9}$$

This has several important implications:

i) The conventional unitary leptonic mixing matrix is replaced by a non-unitary matrix N. One can rewrite Eq. (7) as

$$N = U\left(1 - \frac{1}{2}\sqrt{\bar{m}_\nu}O\mu^{-1}O^\dagger\sqrt{\bar{m}_\nu}\right) \, , \tag{10}$$

which means the inverse seesaw structure yields a characteristic pattern of the unitarity violation. The elements of U are merely rescaled and the non-unitarity effects exhibit correlations with non-trivial experimental implications. Moreover, Eq. (10) justifies the validity of choosing the diagonal basis for charged leptons.

We will use a variant of a convenient parametrization advocated in Ref. [23], namely $N = (1 - \eta)U$, which is particularly suitable for the studies of neutrino oscillation phenomena. The small Hermitian matrix η in the setting of our interest obeys

$$2\eta = FF^\dagger = U\sqrt{\bar{m}}O\mu^{-1}O^\dagger\sqrt{\bar{m}}U^\dagger \, . \tag{11}$$

It is clear that the phases P_ρ do not affect the magnitudes but only the overall phases of the individual entries of η.

ii) The heavy neutrinos n_i and $\tilde{n}_i$ couple to the gauge sector of the SM, and thus, if kinematically accessible, can be produced at hadron colliders. Due to their pseudo-Dirac nature, the lepton number violating collider signatures will be suppressed with respect to the fine-tuned type-I and III scenarios, where the heavy states are Majorana particles. Therefore, if $K \propto F$ is sizeable and the RH sector is accessible, one could expect appreciable rates in the LFV channels.

iii) The LFV decays $\ell_\alpha^- \to \ell_\beta^- \gamma$ are controlled by the magnitude of F that, similar to the fine-tuned type-I case, is not suppressed by the light neutrino masses, and thus can be remarkable [25].

---

[3] Note that this proposition is, indeed, in the spirit of no extra fine-tuning in the neutrino sector. Nevertheless, in what follows, we will consider more general settings as well.

In what follows, we will comment in more detail on the three points above, and focus, in particular, on the discovery potential for the non-unitary effects of the future neutrino oscillation experiments.

IV. PHENOMENOLOGICAL CONSEQUENCES

A. Constraints on non-unitarity parameters

In general, the deviation of the leptonic mixing matrix from unitarity is constrained namely from the universality tests of the weak interactions, rare leptonic decays, invisible width of the Z-boson, and neutrino oscillation data. The current 90 % C.L. bounds on the entries of η are summarized in Refs. [26, 27]: $|\eta_{ee}| < 2.0 \times 10^{-3}$, $|\eta_{\mu\mu}| < 8.0 \times 10^{-4}$, $|\eta_{\tau\tau}| < 2.7 \times 10^{-3}$, $|\eta_{e\mu}| < 3.5 \times 10^{-5}$, $|\eta_{e\tau}| < 8.0 \times 10^{-3}$, and $|\eta_{\mu\tau}| < 5.1 \times 10^{-3}$.

Concerning the shape of O, we will consider three basic situations. First (case I), let O be a unit matrix. Given the correlations of the non-unitarity effects, in particular the simple structure of Eq. (8), the six generic parameters $|\eta_{\alpha\beta}|$ are no longer independent and one can exploit Eq. (11) to improve some of these bounds by almost an order of magnitude. Indeed, Eq. (11) reads in an explicit matrix form

$$\eta \simeq \frac{1}{2}P_\rho\begin{pmatrix} d_1^2 - d_{12}^2 s_{12}^2 & -d_{12}^2 s_{12}c_{12}c_{23} & d_{12}^2 s_{12}c_{12}s_{23} \\ . & d_3^2 - d_{312}^2 c_{23}^2 & d_{312}^2 s_{23}c_{23} \\ . & . & d_3^2 - d_{312}^2 s_{23}^2 \end{pmatrix}P_\rho^\dagger, \tag{12}$$

where $d_{12}^2 \equiv d_1^2 - d_2^2$, $d_{312}^2 \equiv (d_3^2 - d_1^2)s_{12}^2 + (d_3^2 - d_2^2)c_{12}^2$, the small θ_{13} effects have been neglected, and the omitted entries follow from the Hermitian property of η. Examining Eq. (12), the correlations induced in the present framework can be readily obtained, and the current upper bounds on $|\eta_{\alpha\beta}|$ are upgraded to $|\eta_{e\tau}| < 3.5 \times 10^{-5}$, $|\eta_{\mu\tau}| < 8.0 \times 10^{-4}$, $|\eta_{ee}| < 1.6 \times 10^{-3}$, and $|\eta_{\tau\tau}| < 8.0 \times 10^{-4}$.

Second (case II), let μ be flavor blind, i.e., $\mu = \mu_0\mathbb{1}$, and O arbitrary. Equation (11) then yields $\eta = \frac{1}{2}\mu_0^{-1}U\sqrt{\bar{m}_\nu}\exp(2\,\mathrm{i}A)\sqrt{\bar{m}_\nu}U^\dagger$, where A is a real antisymmetric matrix [28]. Following the same strategy, one obtain improved bounds $|\eta_{e\tau}| < 2.3 \times 10^{-3}$ and $|\eta_{\mu\tau}| < 1.5 \times 10^{-3}$, while the other experimental limits are saturated.

Third (case III), one can consider the most general setting relaxing also the degeneracy in the matrix μ. In such a case, all the current experimental bounds can be saturated simultaneously. Nevertheless, this in general does not mean that any configuration of the values of $|\eta_{\alpha\beta}|$ that may be measured in future experiments can be accommodated. However, a detailed analysis of this most generic setting is out of the scope of this work and will be performed elsewhere.

Hence, from the point of view of the future neutrino oscillation experiments, both case-I and II naturally accommodate "sizeable" (i.e., a few per mil) non-unitarity
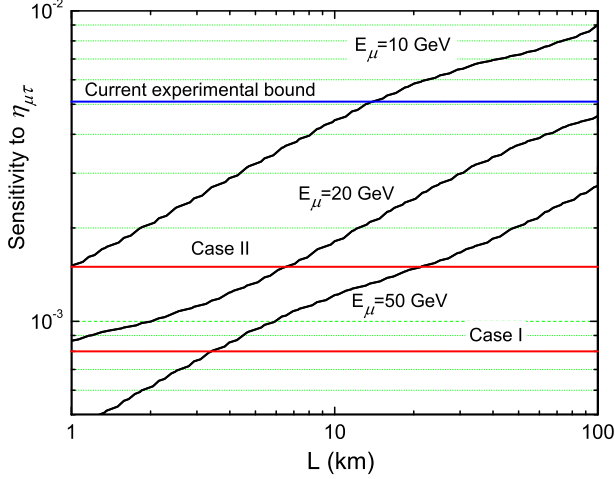
FIG. 1: Sensitivity limits at 90 % C.L. on $\eta_{\mu\tau}$ in the inverse seesaw model as a function of L corresponding to the first two (mandatory) terms in Eq. (16).

effects in the $\nu_\mu \to \nu_\tau$ channel. In principle, this can be used to test the minimal inverse seesaw model.

B. Sensitivity at a neutrino factory

For a non-unitary leptonic mixing matrix N, the vacuum neutrino oscillation transition probability $P_{\alpha\beta}$ can be written as [29]

$$
P_{\alpha\beta} = \sum_{i,j} \mathcal{F}_{\alpha\beta}^i \mathcal{F}_{\alpha\beta}^{j*} - 4 \sum_{i>j} \mathrm{Re}(\mathcal{F}_{\alpha\beta}^i \mathcal{F}_{\alpha\beta}^{j*}) \sin^2 \left(\frac{\Delta m_{ij}^2 L}{4E} \right)
$$
$$
+ 2 \sum_{i>j} \mathrm{Im}(\mathcal{F}_{\alpha\beta}^i \mathcal{F}_{\alpha\beta}^{j*}) \sin \left(\frac{\Delta m_{ij}^2 L}{2E} \right) , \quad (13)
$$

where $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$ are the neutrino mass-squared differences and $\mathcal{F}^i$ are defined by

$$
\mathcal{F}_{\alpha\beta}^i \equiv \sum_{\gamma,\rho} (R^*)_{\alpha\gamma} (R^*)_{\rho\beta}^{-1} U_{\gamma i}^* U_{\rho i} \quad (14)
$$

with the normalized non-unitary factor

$$
R_{\alpha\beta} \equiv \frac{(1-\eta)_{\alpha\beta}}{[(1-\eta)(1-\eta^\dagger)]_{\alpha\alpha}} . \quad (15)
$$

When Earth matter effects are considered, one can replace the vacuum quantities U and m_i by their effective matter counterparts, see e.g. Ref. [30].

As we argued in Sec. IV A, the $\nu_\mu \to \nu_\tau$ channel is the most favorable channel to constrain the model, since it is correlated with $\eta_{\mu\tau}$ which is by far the largest off-diagonal entry of η. In this respect, the best sensitivity is generally provided by short baseline setups, since the standard oscillation effects are suppressed by sines of L in such setups [31, 32, 33].
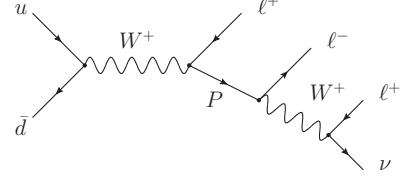

FIG. 2: Scheme of the potentially interesting LHC signature with three charged leptons and missing energy emerging at appreciable rates in the model under consideration.

Thus, in what follows, we will consider the transition probability $P_{\mu\tau}$ for a neutrino factory with a short enough baseline length. We neglect the matter effects, the tiny mixing angle θ_{13}, and the small mass-squared difference Δm_{21}^2. In such a case the transition probability with non-unitary effects reads [31]

$$
P_{\mu\tau} \simeq 4|\eta_{\mu\tau}|^2 + 4s_{23}^2 c_{23}^2 \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \right)
$$
$$
- 4|\eta_{\mu\tau}| \sin \delta_{\mu\tau} s_{23} c_{23} \sin \left(\frac{\Delta m_{31}^2 L}{2E} \right) , \quad (16)
$$

where the last term is CP odd due to the phase $\delta_{\mu\tau}$ of $\eta_{\mu\tau}$, and hence induces distinctive CP-violating effects in neutrino oscillations [23]. Since the model under consideration does not provide any information about $\delta_{\mu\tau}$, we will stick to the most pessimistic scenario with $\delta_{\mu\tau} = 0$, and the non-unitarity effects emerge only from the first "zero distance" term in Eq. (16), which is quadratic in $|\eta_{\mu\tau}|$. For any non-negligible values of $\sin\delta_{\mu\tau}$ one can then expect the non-unitarity effects to be even more pronounced, since the CP-odd contribution is linear in $|\eta_{\mu\tau}|$.

Let us illustrate the feasibility of observing such a signal in a typical neutrino factory setup with an OPERA-like near detector with fiducial mass of 5 kt. We assume a setup with approximately 10^{21} useful muon decays and five years of neutrino running. We make use of the GLoBES package [34, 35] with a slight modification of the template Abstract Experiment Definition Language (AEDL) file for neutrino factory experiments [36, 37]. In Fig. 1, we display the sensitivity to $|\eta_{\mu\tau}|$ as a function of the baseline length L for the near detector. One can observe that such a setup provides indeed an excellent probe for this type of non-unitarity effects. As expected, the sensitivity is decreasing with the baseline length due to the oscillation effects. Thus, a distance $L \lesssim 100$ km would be favorable for the near detector.[4]

---

[4] Note that, practically, an extremely short baseline setup (i.e., $L = 3$ km and $E_\mu = 25$ GeV) may not be efficient, since the beam divergence is not comparable with the size of detector. See Ref. [38] for detailed discussions.

C. Potentially interesting LHC signatures

Since the amount of lepton number violation in the current setting is small (driven by μ) [39], the striking LHC signature of the fine-tuned type-I and III models with like-sign leptons in the final state $pp \to \ell_\alpha^\pm \ell_\beta^\pm + \text{jets}$ [40, 41, 42] is suppressed. Technically, the suppression emerges from the interplay between the graphs with internal lines of the n and $\tilde{n}$ type that tend to cancel due to the opposite CP parities of these states leaving behind only factors proportional to μ. However, the lepton flavor violating processes are insensitive to this effect and in principle one can expect observable signals in the channels with small SM background. For example, one very interesting and prospective channel is the production of three charged leptons and missing energy [43], i.e., $pp \to \ell_\alpha^\pm \ell_\beta^\pm \ell_\gamma^\mp \nu(\bar{\nu}) + \text{jets}$, which is depicted in Fig. 2. Another possible process is the pair production of charged leptons with different flavor and zero missing energy, i.e., $pp \to \ell_\alpha^\pm \ell_\beta^\mp + \text{jets}$. Note that it is difficult to make the observation of this channel at the LHC due to the large SM background [42].

D. Lepton flavor violating decays

The heavy pseudo-Dirac singlets P entering the charged currents due to the non-unitarity effects also contribute to the lepton flavor violating decays $\ell_\alpha \to \ell_\beta \gamma$. The amplitude is proportional to $(FF^\dagger)_{\alpha\beta}$ that measures the amount of non-unitarity in the diagonal sub-blocks of V [44]. In the standard type-I seesaw scenario (i.e., without cancellations), one has approximately $FF^\dagger = \mathcal{O}(m_\nu M_R^{-1})$, and therefore $\text{BR}(\ell_\alpha \to \ell_\beta \gamma) \propto \mathcal{O}(m_\nu^2)$ indicates a strong suppression of LFV decays. However, in the inverse seesaw case, one can have sizeable $F = M_D M_R^{-1}$ in spite of $m_\nu \to 0$. Thus, appreciable LFV rates could be obtained even for strictly massless light neutrinos [45].

V. CONCLUSIONS

In this letter, we have elaborated on the non-unitarity effects in neutrino oscillations due to the relative proximity of the electroweak scale and the scale of the would-be right-handed neutrinos in the inverse seesaw model of light neutrino masses. Unlike the traditional type-I and III seesaw scenarios, this framework does not suffer from naturalness issues even if the heavy sector is low enough to be accessible at the LHC. Moreover, it can accommodate sizeable lepton flavor violating effects.

The simplistic flavor structure of the model, which is argued to be essentially equivalent (in complexity) to the type-I seesaw scenario, yields distinctive correlations between the phenomenological parameters $\eta_{\alpha\beta}$ governing the non-unitarity of the leptonic mixing matrix. In view of the possible significant off-diagonal entry $\eta_{\mu\tau}$, we have studied the discovery potential of a neutrino factory experiment with an OPERA-like near detector in the $\mu \to \tau$ channel and presented the relevant sensitivity limits to $\eta_{\mu\tau}$. Potentially interesting signatures at the LHC and in the lepton flavor violating decays have also been discussed.

[1] P. Minkowski, Phys. Lett. **B67**, 421 (1977).

[2] T. Yanagida, in *Proc. Workshop on the Baryon Number of the Universe and Unified Theories*, edited by O. Sawada and A. Sugamoto (1979), p. 95.

[3] R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980).

[4] J. Schechter and J. W. F. Valle, Phys. Rev. **D22**, 2227 (1980).

[5] G. Lazarides, Q. Shafi, and C. Wetterich, Nucl. Phys. **B181**, 287 (1981).

[6] R. N. Mohapatra and G. Senjanović, Phys. Rev. **D23**, 165 (1981).

[7] R. Foot, H. Lew, X. G. He, and G. C. Joshi, Z. Phys. **C44**, 441 (1989).

[8] S. Antusch, C. Biggio, E. Fernández-Martínez, M. B. Gavela, and J. López-Pavón, JHEP **10**, 084 (2006), hep-ph/0607020.

[9] W. Buchmüller and C. Greub, Nucl. Phys. **B363**, 345 (1991).

[10] A. Pilaftsis, Z. Phys. **C55**, 275 (1992), hep-ph/9901206.

[11] G. Ingelman and J. Rathsman, Z. Phys. **C60**, 243 (1993).

[12] C. A. Heusch and P. Minkowski, Nucl. Phys. **B416**, 3 (1994).

[13] J. Kersten and A. Y. Smirnov, Phys. Rev. **D76**, 073005 (2007), arXiv:0705.3221.

[14] B. Bajc and G. Senjanović, JHEP **08**, 014 (2007), hep-ph/0612029.

[15] B. Bajc, M. Nemevsek, and G. Senjanović, Phys. Rev. **D76**, 055011 (2007), hep-ph/0703080.

[16] P. Fileviez Perez, T. Han, G.-y. Huang, T. Li, and K. Wang, Phys. Rev. **D78**, 015018 (2008), arXiv:0805.3536.

[17] M. Malinský, T. Ohlsson, and H. Zhang, Phys. Rev. **D79**, 011301(R) (2009), arXiv:0811.3346.

[18] R. N. Mohapatra and J. W. F. Valle, Phys. Rev. **D34**, 1642 (1986).

[19] T. Appelquist and R. Shrock, Phys. Lett. **B548**, 204 (2002), hep-ph/0204141.

[20] T. Appelquist, M. Piai, and R. Shrock, Phys. Rev. **D69**, 015002 (2004), hep-ph/0308061.

[21] K. Kanaya, Prog. Theor. Phys. **64**, 2278 (1980).

[22] J. Schechter and J. W. F. Valle, Phys. Rev. **D25**, 774 (1982).

[23] G. Altarelli and D. Meloni, Nucl. Phys. **B809**, 158 (2009), arXiv:0809.1041.

[24] J. A. Casas and A. Ibarra, Nucl. Phys. **B618**, 171 (2001), hep-ph/0103065.

[25] F. Deppisch and J. W. F. Valle, Phys. Rev. **D72**, 036001 (2005), hep-ph/0406040.

[26] S. Antusch, J. P. Baumann, and E. Fernández-Martínez, Nucl. Phys. **B810**, 369 (2009), arXiv:0807.1003.

[27] S. Antusch, M. Blennow, E. Fernández-Martínez, and J. López-Pavón (2009), arXiv:0903.3986.

[28] S. Pascoli, S. T. Petcov, and C. E. Yaguna, Phys. Lett. **B564**, 241 (2003), hep-ph/0301095.

[29] T. Ohlsson and H. Zhang, Phys. Lett. **B671**, 99 (2009), arXiv:0809.4835.

[30] D. Meloni, T. Ohlsson, and H. Zhang, JHEP (to be published), arXiv:0901.1784.

[31] E. Fernández-Martínez, M. B. Gavela, J. López-Pavón, and O. Yasuda, Phys. Lett. **B649**, 427 (2007), hep-ph/0703098.

[32] S. Goswami and T. Ota, Phys. Rev. **D78**, 033012 (2008), arXiv:0802.1434.

[33] A. Donini, K.-i. Fuki, J. López-Pavón, D. Meloni, and O. Yasuda (2008), arXiv:0812.3703.

[34] P. Huber, M. Lindner, and W. Winter, Comput. Phys. Commun. **167**, 195 (2005), hep-ph/0407333.

[35] P. Huber, J. Kopp, M. Lindner, M. Rolinec, and W. Winter, Comput. Phys. Commun. **177**, 432 (2007), hep-ph/0701187.

[36] D. Autiero et al., Eur. Phys. J. **C33**, 243 (2004), hep-ph/0305185.

[37] P. Huber, M. Lindner, M. Rolinec, and W. Winter, Phys. Rev. **D74**, 073003 (2006), hep-ph/0606119.

[38] J. Tang and W. Winter (2009), arXiv:0903.3039.

[39] W.-Y. Keung and G. Senjanović, Phys. Rev. Lett. **50**, 1427 (1983).

[40] A. Datta, M. Guchait, and A. Pilaftsis, Phys. Rev. **D50**, 3195 (1994), hep-ph/9311257.

[41] T. Han and B. Zhang, Phys. Rev. Lett. **97**, 171804 (2006), hep-ph/0604064.

[42] F. del Aguila, J. A. Aguilar-Saavedra, and R. Pittau, JHEP **10**, 047 (2007), hep-ph/0703261.

[43] F. del Aguila and J. A. Aguilar-Saavedra, Phys. Lett. **B672**, 158 (2009), 0809.2096.

[44] A. Ilakovac and A. Pilaftsis, Nucl. Phys. **B437**, 491 (1995), hep-ph/9403398.

[45] J. Bernabeu, A. Santamaria, J. Vidal, A. Mendez, and J. W. F. Valle, Phys. Lett. **B187**, 303 (1987).